EXPERIMENTAL STUDY OF THE DEFORMATION AND FRACTURE OF CIRCULAR ALUMINUM PLATES UNDER THE INFLUENCE OF A SHOCK WAVE

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The behavior of plates under the influence of strong shock waves is important in relation to the study of large plastic deformations and fracture. As a rule, large plastic deformations and fracture of plates are treated separately in the majority of existing papers, which are surveyed in [1-4]. In the present article we report an experimental study of the dependence of the buckling deflection of aluminum plates of various diameters and thicknesses on the impulsive load created by the detonation of a flat high-explosive charge in a shock tube. We establish the critical buckling deflections responsible for fracture of the plates. The results are presented in generalized form.

Plates used as diaphragms for shock tubes are assumed to be edge-supported by clamping between two tightly bolted flanges. When the high-pressure and low-pressure chambers have different diameters, the plate is clamped around the larger diameter, i.e., the plate is impulsively loaded over part of its surface. The plate is assumed to be loaded axisymmetrically and uniformly in either case.

Our experiments were carried out on shock tubes with a plane shock wave generated by the detonation of a flat laminated high-explosive charge under standard atmospheric conditions [5]. The shock tubes were steel cylinders with inside diameters d = 0.09 m, 0.19 m, and 0.40 m and with lengths L = 0.5 m and 2 m. The explosive charge was placed in the cross section located at equal distances from the ends of the tube. The impulsive load on the plate was varied by changing the thickness of the charge. The impulsive load was determined ballistically from the launching of a heavy, nondeformable target placed at the site of the investigated plate, and the waveform of the shock wave reflected from the nondeformable rigid wall was determined by a piezoelectric transducer [5]. The duration of the pulse depended on the distance x from the charge to the loaded object and was equal to 10^{-4} sec and $5\cdot10^{-4}$ sec at the 0.1-peak pressure level for the selected distances of 0.25 m and 1 m, respectively.

The buckling deflection δ at the center of the plate was determined experimentally as a function of the specific impulse i for D-16T aluminum alloy plates. The plates had thicknesses $h = (1\text{-}4)\cdot10^{-3}$ m, and the diameter of the clamping flanges was varied from 0.09 m to 0.63 m. Figure 1 shows photographs of the deformation process of a plate with a diameter of 0.19 m, which were taken with a fast-framing camera and which exhibit the motion of the plastic link from the clamped edge toward the center of the plate. The time between consecutive frames is $66\cdot10^{-6}$ sec, and the velocity of the plastic link is ~400 m/sec. A diagram of the motion of the polar point for a plate with d = D = 0.4 m and $h = 4\cdot10^{-3}$ m with loading by an explosive charge of thickness $\Delta = 5\cdot10^{-3}$ m at a distance x = 0.25 m is shown in Fig. 2. The dome corresponding to permanent set of the plate is formed immediately prior to stopping of the polar point, where the maximum deflection at the center is slightly greater than the buckling deflection. The maximum velocity of the polar point for the experiment represented in Fig. 2 is 260 m/sec, and the dashed curve gives an estimate of the motion of the free plate without the influence of clamping under the influence of a pressure pulse corresponding to the reflected shock wave.

The buckling deflection as a function of the impulsive load for some of the investigated plates is plotted in Fig. 3, in which the curves are enumerated according to the following conditions: 1) d = D = 0.40 m, $h = 1.5\cdot10^{-3}$ m; 2) d = D = 0.40 m, $h = 4\cdot10^{-3}$ m; 3) d = 0.09 m, D = 0.30 m, $h = 4\cdot10^{-3}$ m; 4) d = D = 0.09 m, $h = 4\cdot10^{-3}$ m. All the $\delta(i)$ curves

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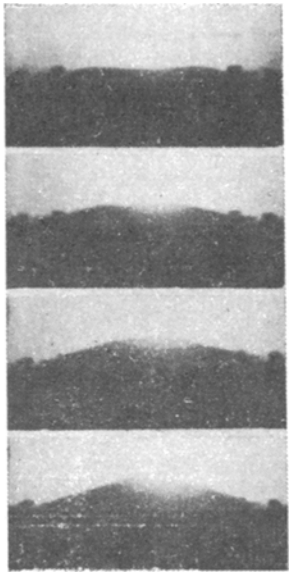

Fig. 1

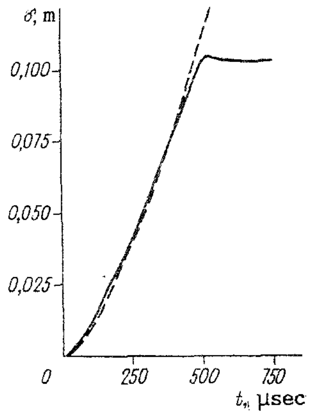

Fig. 2

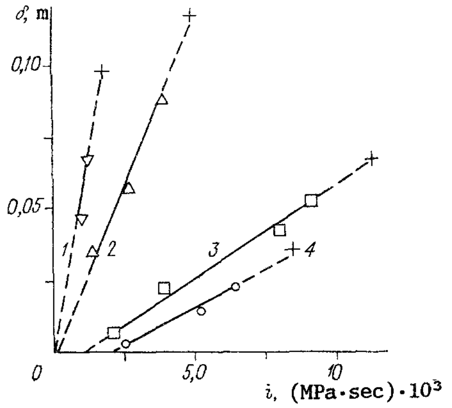

Fig. 3

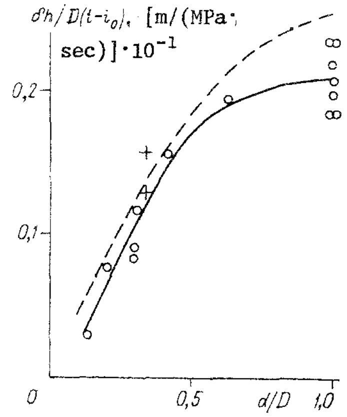

Fig. 4

in the figure are linear up to fracture of the plate, which is indicated by the plus symbol. The initial specific impulses i_0 leading to buckling are determined by extrapolation of the linear dependence $\delta(i)$; they fall in the interval from $i_0 = 0$ to 0.002 MPa·sec and correspond to the empirical function $i_0 = K_0(h/d^2)\tau\sqrt{d/D}$ ($K_0 = 40$ MPa·m; h, and d are in units of 10^{-2} m, τ in sec). For plates loaded over the entire surface (d = D) the total impulse leading to plastic deformation of the plate depends only on its thickness and the duration of the load.

The majority of the data on the deformation and fracture of plates have been obtained for impulsive loads with a duration of 10^{-4} sec. A comparison of the experimental results confirms the fact that the buckling deflections of the plates not only depend linearly on the impulse, but are also proportional to the parameters D and 1/h. The results can therefore be represented by the empirical function $\delta h/D(i - i_0) = f(d/D)$ plotted in Fig. 4, where the pulses represent previous data [6] corresponding to the impulsive loading of aluminum alloy plates by the detonation of a laminated charge through a polyethylene buffer layer.

We now compare the reported data with the calculated values of the buckling deflection. The dissipation of energy in the plate material is entirely attributable to effects of membrane forces in the range $\delta \gg h$ [4]. We assume that the material is inelastoplastic, and we investigate the loading of a freely supported plate by a rectangular pressure pulse $i = p_0\tau$ (p_0 is the pressure, and τ is the duration). In this case the work done by the pressure forces is spent in kinetic energy and in irreversible elongation of the centroidal surface of the plate:

$$\int_0^{d/2} p_0 w(r, t)\, 2\pi r\, dr = \rho h \int_0^{D/2} \left(\frac{\partial w}{\partial t}\right)^2 \pi 2\, dr + \sigma_0 h \int_0^{D/2} \left(\frac{\partial w}{\partial r}\right)^2 \pi r\, dr.$$

Here r is the radial distance, t is the time, w(r, t) is the bending deflection of the centroidal surface, and ρ and σ_0 are the density and yield point of the material. The kinetic energy of transverse motion can be disregarded under these conditions [1]. We seek w in the form [4] $w(r, t) = \sum_1^\infty \delta_n(t) J_0\left(\alpha_n \frac{2r}{D}\right)$ $\left(\sum_{n=1}^\infty \delta_n(t)\right.$ is the bending deflection at the center of the plate, $J_0(\alpha_n \cdot 2r/D)$ is the zeroth-order Bessel function of the first kind, and α_n are the roots of the equation $J_0(\alpha_n) = 0$, which is obtained from the boundary condition for the freely supported plate $w(D/2, t) = 0$). Taking these conditions into account, we have the equation of motion

$$\dot{\delta}_n^2 = \frac{4p_0 d}{\rho h D} \frac{J_1\left(\alpha_n \frac{d}{D}\right)}{\alpha_n J_1^2(\alpha_n)} \delta_n - \frac{4\sigma_0 \alpha_n^2}{\rho D^2} \delta_n^2, \quad 0 \leqslant t \leqslant \tau, \quad \delta_n(t = 0) = 0,$$

from which it follows that

$$w(r, t) = \sum_{n=1}^\infty \frac{B_n}{\omega_n^2}(1 - \cos \omega_n t) J_0\left(\alpha_n \frac{2r}{D}\right), \tag{1}$$

where

$$B_n = \frac{2p_0 d}{\rho h D} \frac{J_1(\alpha_n d/D)}{\alpha_n J_1^2(\alpha_n)}; \quad \omega_n = \sqrt{\frac{\sigma_0}{\rho}} \frac{2\alpha_n}{D};$$

and $J_1(\ )$ denotes the first-order Bessel function of the first kind.

At $t > \tau$, i.e., when external forces are absent, the total energy of the plate is an integral of motion. This gives us the equation for free vibrations in the plastic domain $\ddot{\delta}_n + \omega_n^2 \delta_n = 0$; solving this equation subject to initial conditions determined from Eq. (1), for $t = \tau$, we obtain

$$w(r, t) = \sum_{n=1}^\infty \frac{B_n}{\omega_n^2} [(1 - \cos \omega_n \tau) \cos \omega_n (t - \tau) + \sin \omega_n \tau \cdot \sin \omega_n (t - \tau)] J_0\left(\alpha_n \frac{2r}{D}\right).$$

The plate stopping time is determined from the condition $\partial w/\partial t = 0$ and is equal to the maximum of the times t_n representing the roots of the equation $\tan \omega_n(t_n - \tau) = \sin \omega_n \tau/(1 - \cos \omega_n \tau)$. When the pulse duration is much shorter than the natural period of the plate in the plastic domain, $\tau \ll 2\pi/\omega_1 = T_1$, the stopping time is $t_1 = T_1/4$, and the buckling deflection at the center is

$$\delta = \frac{id}{h\sqrt{\rho \sigma_0}} \sum_{n=1}^\infty \frac{J_1(\alpha_n d/D)}{\alpha_n^2 J_1^2(\alpha_n)}. \tag{2}$$

As the pulse duration is increased from $\tau = 0$ to $\tau = T_n/2$, the deflection of the corresponding harmonic decreases by approximately 40%, and the stopping time increases linearly from $T_1/4$ to $T_1/2$. The most efficient loading occurs when the duration $\tau < T_1/4$. The behavior of the parameter $\delta h/iD$, calculated according to Eq. (2) for $\sigma_0 = 3.7 \cdot 10^2$ MPa [3] and represented by the dashed curve in Fig. 4, is observed to be somewhat higher than the experimental data, but by no more than 20%. Despite a certain indeterminacy in the experimental boundary conditions, this agreement can be deemed satisfactory. It is particularly accurate for plates of large diameter (D = 0.4 m). This is probably attributable to the fact that, according to [3], the clamping conditions do not play a significant role for thin plates of large diameter.

An increase in the pulse duration from 10^{-4} sec to $5 \cdot 10^{-4}$ sec is accompanied by a certain decrease in the buckling deflections in the reported experiments, at least for large-diameter plates. According to the experimental data, the parameter $\delta h/(i - i_0)$ decreases by 18% for plates of diameter 0.4 m. This result is attributable to the fact that the period of the first plastic harmonic of the plate, $T_1 \sim 10^{-3}$ sec, exceeds the indicated range of durations.

It is interesting to note that the quantity $\sqrt{\sigma_0/\rho}$ with the units of velocity in the expression for the plastic harmonic frequencies ω_n is approximately equal to 370 m/sec for

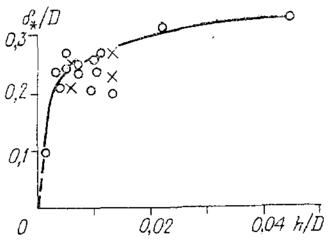

Fig. 5

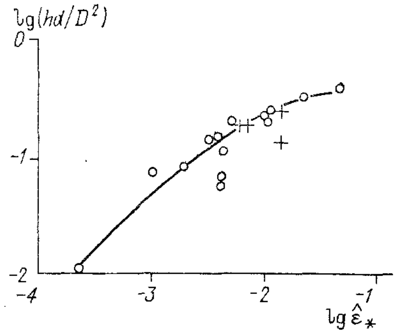

Fig. 6

the above-stated value of σ_0, in good agreement with the measured velocity of the plastic link v ~ 400 m/sec.

Two types of shock-induced fracture of the plates were observed in the experiments described here: The material fractured along the clamped edge of the plate when subjected to loading of the entire surface (d/D = 1), whereas radial cracks appeared at the center in the case d/D < 1.

An analysis of the data shows that the critical buckling deflections δ_* corresponding to fracture of the plates can be represented by the generalized functional relation $\delta_*/D = \phi(h/d)$ (Fig. 5), which is valid for all values of d/D, i.e., for any type of fracture. The quantity δ_* was considered to be the average of the maximum buckling deflection at which fracture did not set in and the minimum value at which the plate fractured. The pluses in Fig. 5 represent the experimental results of [1] for the loading of aluminum plates by an underwater explosion. The graph indicates that δ_* is practically independent of h for thick plates (h/D > 0.02). We note that the observed character of the fracture is qualitatively consistent with earlier predictions [3].

We estimate the average strains $\hat{\varepsilon}$, assuming that they are associated entirely with tension of the plate [7]:

$$\hat{\varepsilon} = \ln \sqrt{\frac{S}{S_0}} \approx 2\int_0^{D/2} \left(\frac{\partial w}{\partial r}\right)^2 \pi r\, dr/\pi D^2$$

(S_0 and S are the initial and instantaneous areas of the centroidal surface of the plate). Using Eq. (2), we obtain

$$\hat{\varepsilon} = \left(\frac{\delta}{D}\right)^2 \sum_{n=1}^{\infty} \frac{J_1^2(\alpha_n d/D)}{\alpha_n^2 J_1^2(\alpha_n)} \Bigg/ \left[\sum_{n=1}^{\infty} \frac{J_1(\alpha_n d/D)}{\alpha_n^2 J_1^2(\alpha_n)}\right]^2. \quad (3)$$

An estimation according to this equation shows that, for example, that the average strain for plates of diameter d = D = 0.19 m and thickness $h = 2\cdot10^{-3}$ m at δ = 0.03 m is $\hat{\varepsilon} \approx 0.085$. Etch-line measurements in one of the tests showed that the maximum radial strain is attained at the center and has a value ~0.1, and the average strain can be assumed equal to 0.06. Equation (3) thus gives satisfactory estimates and can be used to find the average fracture strain $\hat{\varepsilon}_*$ when the experimental values of δ_* are used. This type of function $\hat{\varepsilon}_* = F(hd/D^2)$ is plotted in Fig. 6 (the pluses represent the experimental values of the radial fracture strains at the center of aluminum plates exposed to an underwater explosion [1]). The smallest average critical tensile strain leading to fracture of the plate occurred for d = 0.09 m, D = 0.63 m, $h = 1\cdot10^{-3}$ m and had a value $\hat{\varepsilon}_* \approx 0.01$. The maximum value $\hat{\varepsilon}_* = 0.38$ corresponded to loading of a plate with d = D = 0.09 m and $h = 4\cdot10^{-3}$ m.

The results of the present study can be used in the selection of diaphragms for shock tubes.

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GROWTH OF DISTURBANCES IN A SUPERSONIC BOUNDARY LAYER

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The onset of turbulence in supersonic flows has stimulated investigations of the stability of compressible boundary layers. The first theoretical studies of this problem were reported by Lees, Lin, and Dunn (see Lin [1]). Attempts to verify the theory experimentally have been undertaken [2, 3], but the experiments were performed with natural disturbances, whose wave spectra were not controllable. Consequently, although spatially growing disturbances were successfully observed in [3], the comparison with the theoretical results was of a qualitative nature. The outcome in [2], on the other hand, proved essentially unsuccessful. More reliable experiments are reported by Kendall [4], who succeeded in confirming the theory in application to two-dimensional second-mode disturbances and three-dimensional (oblique) waves at a Mach number M = 4.5 and a Reynolds number $\mathrm{Re} = \sqrt{U_\infty x/\nu_\infty} = 1550$. The causes of the failures in studies of two-dimensional first-mode disturbances have yet to be explained.

Experimental studies of the stability of a supersonic boundary layer have been carried out at the Institute of Theoretical and Applied Mechanics of the Siberian Branch of the Academy of Sciences of the USSR [5]. Reliable data were obtained with the use of controllable artificial disturbances. They fully corroborate the basic principles of the theory of the stability of both plane-parallel [1] and slightly nonparallel [6] compressible flows. It has been established [7] that the wave number spectrum contains several maxima at a given frequency. The principal maximum corresponds to the results of the linear theory. The others could not be explained within the scope of the existing theory. The upstream incursion of disturbances has been observed in later experiments [8], but has not been investigated theoretically. Moreover, the spatial growth rates of waves whose fronts propagate at an angle $\chi < 45°$ relative to the main flow differ from those predicted by the theory of plane-parallel flows. In the present study, therefore, we continue the theoretical investigation of the growth of disturbances in a supersonic boundary layer, taking the new experimental data into account.

1. The stability of a supersonic boundary layer on a flat plate is analyzed both in the parallel-flow approximation and with allowance for departures from parallelism. The truncated Dunn–Lin equations (see [9]) are used in the first case, and the theory of [10] is used in the second case. In the calculations it is assumed that M = 4.0, $\mathrm{Re} = \sqrt{U_\infty x/\nu_\infty} = 600$, the Prandtl number Pr = 0.72, and the adiabatic exponent $\gamma = 1.4$. The viscosity–temperature relation is described by Sutherland's formula. Here U_∞ and ν_∞ are the velocity and viscosity at the outer edge of the boundary layer.

The disturbance is assumed to be a function of the dimensionless coordinates and time in the form

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